LITERATURE CITED

1. L. V. Al'tshuler, "Phase transformations in shock waves," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1978).
2. M. A. Podurets, G. V. Simakov, and R. F. Trunin, "Phase equilibrium in a shock-compressed quartz and the character of the kinetics of the phase transition," Izv. Akad. Nauk SSSR, Fiz. Zemlyi, No. 7 (1976).
3. M. A. Podurets and R. F. Trunin, "A property of shock compressibility of quartzite," Dokl. Akad. Nauk SSSR, 195, No. 4 (1970).
4. A. Keh and S. Weissmann, in: Proc. of the Conf. on Electron Microscopy and Strength of Materials, New York (1963).
5. J. Gilman, "Dislocation dynamics and the response of materials," Appl. Mech. Rev., 21, No. 8 (1968).
6. E. H. Lee and D. T. Liu, "Finite-strain elastic-plastic theory with application to plane-wave analysis," J. Appl. Phys., 38, No. 1 (1967).
7. E. H. Lee, "Elastic-plastic deformation at finite strain," J. Appl. Mech., 36, No. 1 (1969).
8. B. Ya. Lyubov, Kinetic Theory of Phase Transformations [in Russian], Metallurgiya, Moscow (1969).

THE PROCESS OF SPALL FRACTURE

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Many investigations of spall phenomena after the emergence of the compression pulse to the free surface of a specimen show that the strength value realized in spalling depends on the characteristic time of action of the load. A number of studies [1-4] propose discrete criteria for spall fracture which determine the possibility of failure in terms of the value of the tensile stress and the time during which it acts at a particular cross section of the specimen. However, on the one hand, the load at any cross section may, in general, vary arbitrarily, and on the other hand, the failure process itself leads to a drop in the tensile stress, which makes the actual application of the discrete spall criteria difficult. The authors of [5-7] discuss the possibility of introducing a continuous measure of failure into the spall criterion; such a measure may be the dimensions and number of the cracks in the specimen, the residual strength of a half-ruptured specimen, etc. Experimental information on the failure process can be obtained from a metallographic analysis of preserved specimens [5, 6], or from experiments on the continuous recording of the velocity of the free surface of the specimen when a compression pulse and a "spall" pulse emerge onto it [8-11]. It is impossible at the present time to obtain continuous quantitative information directly from the failure zone.

In the present article we consider the effect of the kinetics of failure on the gas dynamics of a wave process. In the gas-dynamic analysis of a phenomenon, it is most convenient to use the specific volume of a crack, v_{cr}, as the measure of the failure. The shear strength of the medium will be disregarded in what follows. The rate of growth of the cracks (or pores), as can be deduced from general considerations [6, 7], is determined by the value of the negative pressure p acting on the material and by the degree of failure achieved, v_{cr}:

$$\dot{v}_{cr} = f(p,\ v_{cr}). \tag{1}$$

Barbee et al. [6] have proposed specific expressions for the failure kinetics of (1), which are based on a model of exponential generation and ductile growth of the cracks.

In order to see what kind of information concerning the effect of continuous failure on the gas dynamics of the process can be obtained in general form, we shall follow the change of state of a substance along the characteristics in a linear material, i.e., in a material whose equation of state has the form

$$\frac{\rho^2}{\rho_0^2}\left(\frac{\partial p}{\partial \rho}\right)_{v_{cr}=\mathrm{const}} = a^2 = \mathrm{const}, \tag{2}$$

where ρ, ρ_0 are the instantaneous and initial values of the density of the substance. The specific volume of the failed medium, v, consists of the volume of the solid material, v_{sol}, and the volume of the cracks, v_{cr}:

$$v = v_{sol} + v_{cr}. \tag{3}$$

The equations of motion and continuity, taking account of (1)-(3), for a one-dimensional case have in Lagrangian coordinates the form

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$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \quad \frac{\partial p}{\partial t} + \frac{1}{\rho_0 a^2} \frac{\partial u}{\partial h} - \rho_0^2 a^2 \frac{\partial v_{cr}}{\partial t} = 0, \tag{4}$$

where u is the mass velocity of the substance, h is a Lagrangian coordinate.

To determine the characteristic directions, we express the derivatives with respect to time in the system of equations (1)-(4), which describe the motion of the failed medium in terms of the derivatives d/dt in the direction dh/dt = λ:

$$\begin{aligned} &\frac{1}{\rho_0} \frac{\partial p}{\partial h} - \lambda \frac{\partial u}{\partial h} = -\frac{du}{dt}, \\ &\lambda \frac{\partial p}{\partial h} - \rho_0 a^2 \frac{\partial u}{\partial h} - \rho_0^2 a^2 \lambda \frac{\partial v_{cr}}{\partial h} = \frac{dp}{dt} - \rho_0^2 a^2 \frac{dv_{cr}}{dt}, \\ &\lambda \frac{\partial v_{cr}}{\partial h} = \frac{dv_{cr}}{dt} - f(p, v_{cr}). \end{aligned} \tag{5}$$

By definition, the selected direction λ will be characteristic in the case when the determinant of the system (5) vanishes. From this we determine $\lambda = \pm a$, $\lambda = 0$, and consequently the characteristics in the case under consideration, as in a nonrelaxing medium, are straight lines with inclination $dh/dt = \pm a$ and a particle trajectory h = const.

The derivatives of the mass velocity and the motions along the characteristics C_+, taking (2)-(4) into consideration, have the form

$$\begin{aligned} &\left.\frac{du}{dt}\right|_{C_+} = \frac{\partial u}{\partial t} - \frac{1}{\rho_0 a} \frac{\partial p}{\partial t} + \rho_0 a \frac{\partial v_{cr}}{\partial t}, \\ &\left.\frac{dp}{dt}\right|_{C_+} = \frac{\partial p}{\partial t} - \rho_0 a \frac{\partial u}{\partial t} = -\rho_0 a \left.\frac{du}{dt}\right|_{C_+} + \rho_0^2 a^2 \dot{v}_{cr}. \end{aligned} \tag{6}$$

Analogously, along the characteristics C_-,

$$\left.\frac{dp}{dt}\right|_{C_-} = \rho_0 a \left.\frac{du}{dt}\right|_{C_-} + \rho_0^2 a^2 \dot{v}_{cr} \tag{7}$$

From (6), (7) it can be seen that when $\dot{v}_{cr} > 0$, the trajectories of the change of state along the characteristics in the coordinates p, u deviate from the straight lines $p = \pm \rho_0 a u + \text{const}$, defined by the Riemann invariants, in the direction of higher pressure.

We consider the variation of the amplitude of a tension wave after reflection from a free surface of a triangular compression pulse being propagated in a positive direction. We shall indicate by + the states immediately in front of the tension jump, and by − the states immediately behind the jump. For a rarefaction jump being propagated in a negative direction, the Rankine–Hugoniot condition is satisfied:

$$p^- - p^+ = -\rho_0 a (u^- - u^+). \tag{8}$$

Bearing in mind that the reflected wave is superimposed on the incident simple compression wave, for which $\dot{v}_T = 0$ and $dp = \rho_0 a du$, we find from (7), (8) that

$$\left.\frac{dp}{dt}\right|_{C_-^-} = 2 \left.\frac{dp}{dt}\right|_{C_-^+} - \left.\frac{dp}{dt}\right|_{C_-^-} + \rho_0^2 a^2 \dot{v}_{cr} = 2\dot{p}_0 + \frac{1}{2} \rho_0^2 a^2 \dot{v}_{cr0}, \tag{9}$$

where $\dot{p}_0 = \frac{1}{2} \left.\frac{dp}{dt}\right|_{C_-^+} = \left(\frac{\partial p}{\partial t}\right)_h$ is the rate of change of the pressure in the incident pulse at the moment when the tension jump approaches the given particle; $\dot{v}_{cr0}$ is the initial rate of failure immediately behind the jump. According to (9), when $\dot{v}_{cr0} > 0$, the growth in the amplitude of the tension wave takes place more slowly than in the case of an unfailed medium. This deduction must be taken into consideration when we determine the spall strength of materials. As an example, we shall consider the case when the rate of failure is a linear function of the pressure:

$$\dot{v}_{cr0}(p) = -2Ap^-/\rho_0^2 a^2.$$

For an incident triangular pulse ($\dot{p}_0$ = const) we can integrate (9) to obtain an expression for the amplitude of the tension wave

$$p^- = 2\dot{p}_0 [1 - \exp(-At)]/A = 2\dot{p}_0 \left[1 - \exp\left(A \frac{h - h_0}{a}\right)\right] \Big/ A, \tag{10}$$

where h_0 is the coordinate of the free surface of the specimen. According to (9), (10), the amplitude of the tension wave asymptotically approaches a value determined by the condition

$$\dot{v}_{cr0} = -4\dot{p}_0/\rho_0^2 a^2, \quad p^- = 2\dot{p}_0/A.$$

Information on spalling was obtained on the free surface of a specimen in the form of a compression wave (a spall pulse) resulting in an increase of the experimentally recorded velocity of the surface, w(t). We shall consider the conditions on the characteristics which correspond to a minimum on the profile of w(t).

It is obvious that the minimum on the profile of w(t) corresponds to a confluence of the trajectories of the change of state along the C_+ characteristics. It can be shown that when $\dot{v}_{cr} > 0$, at a point of confluence the trajectories of the change of state along the characteristics C_+ and C_- have a common tangent. To see this, we note that otherwise when the trajectory of the change of state along the characteristic C_- passes through a point of confluence the conditions $\left.\frac{dp}{dt}\right|_{C_-} = 0$ and $\left.\frac{du}{dt}\right|_{C_-} = 0$ must be satisfied, and according to (7), this is possible only when $\dot{v}_{cr} = 0$. Taking account of the fact that $\left.\frac{du}{dt}\right|_{C_+} + \left.\frac{du}{dt}\right|_{C_-} = 2\frac{\partial u}{\partial t}$ and $\left.\frac{dp}{dt}\right|_{C_+} + \left.\frac{dp}{dt}\right|_{C_-} = 2\frac{\partial p}{\partial t}$, we arrive at the conclusion that at the point of confluence the trajectories of the change of state for a fixed particle along the characteristics C_+ and C_- have a common tangent.

Further investigation of the process was carried out by numerical simulation of the phenomenon. Using the finite-difference method, we solved on a computer the one-dimensional gasdynamics problem of the flow in a failed plate after the emergence of a compression pulse of triangular or square shape onto the free surface of the plate. The problem was solved in the acoustic approximation [an equation of state in the form (2)] and the hydrodynamic approximation. In the acoustic approximation the problem was solved by the method of characteristics; the calculation step with respect to time was taken equal to $\Delta t = \Delta h/a$, where Δh was the calculation step in the spatial dimension. The length of the incident compression pulse was taken equal to the thickness of the plate. The solution in the hydrodynamic approximation was obtained by the transverse method using a checkerboard network and quadratic pseudoductility [12]. The equation of state was given in the form

$$p = \rho_0 c_0^2 \left[\exp\left(4b \frac{v_0 - v_{sol}}{v_0}\right) - 1 \right] \Big/ 4b,$$

where $v_0 = 1/\rho_0$; c_0, b are constants in the linear approximation of the shock-wave adiabat of the substance $D = c_0 + bu$.

The calculation of the change of state at each time layer in both cases was carried out in two stages, in the first of which we assumed that the failure process was frozen, while in the second we froze the gasdynamic process and carried out the calculation of the failure with constant v, u. The new pressure values resulting from the failure were taken as the initial values for the following gas-dynamic step. The density and compressibility of the investigated medium were taken to correspond to iron. The failure kinetics (1) were varied.

Some of the calculations were carried out by using the kinetics of the ductile growth of cracks [6]:

$$\dot{v}_{cr} = -p[v_{cr} + \dot{v}_{cr0}(p)]/k, \tag{11}$$

where k is a constant similar in dimension and physical meaning to ductility, $v_{cr0}(p)$ is a function of pressure which determines the initial rate of failure and which can be treated as the rate of generation of centers of failure or the initial distribution of potential centers of failure in the material. We carried out calculations with two types of functions $v_{cr0}(p)$:

$$v_{cr0} = v^* \exp(p/p^*); \tag{12}$$

$$v_{cr0} = v^* \left[1 + th\left(\frac{p - p_k}{p^*}\right)\right]. \tag{13}$$

Figure 1 shows the results of the calculations in the acoustic approximation for the velocity w(t) of the free surface of the plate for the case of failure kinetics in the form (11), (12) with constants $k = 2 \cdot 10^3\ g \cdot cm^{-1} \cdot sec^{-1}$, $v^* = 10^{-13}\ cm^3 \cdot g^{-1}$, $p^* = -1 \cdot 10^9\ g \cdot cm^{-1} \cdot sec^{-2}$. Qualitatively the calculated profiles are found to be similar to those recorded experimentally [8-11]; we observe an increase in the value of w(t), due to the emergence onto the surface of information concerning the spall, and observe subsequent damped oscillations in the velocity w in the process of reverberation of the spall pulse. Typical profiles of the pressure p(t) in the failure zone are shown in Fig. 2. In the same figure, using the same time scale, we show the profiles for the velocity of the free surface, w(t), and the specific volume of the cracks, $v_{cr}(t)$, for the case $k = 1.25 \cdot 10^3$, $v^* = 3 \cdot 10^{-14}$, $p^* = -1 \cdot 10^9$. The numbers next to the curves indicate the distance l (in millimeters) from the free surface of the specimen. The failure process leads to a decrease in the value of the negative pressure. Figure 3 shows the distribution curves for the degree of failure, $v_{cr}(l)$, over the thickness of the specimen, calculated for the same failure-kinetics constants. The numbers next to the curves correspond to the interval of time (in microseconds) from the moment when the compression-pulse front emerges onto the free surface of a plate 4 mm thick. It can be seen that calculations using the kinetics of (11), (12) yield a blurred failure region. Figure 4 shows part of the p–u diagram, obtained by

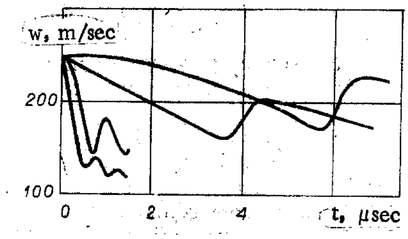

Fig. 1

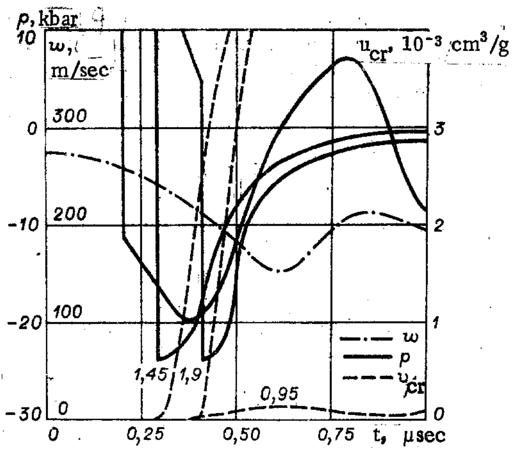

Fig. 2

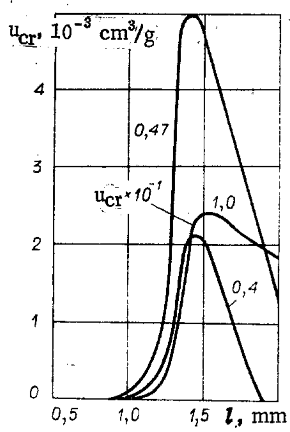

Fig. 3

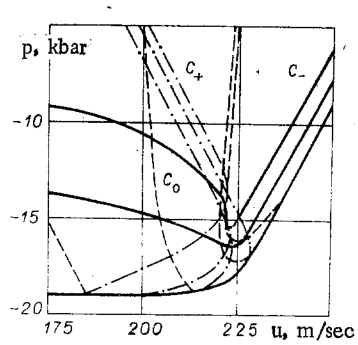

Fig. 4

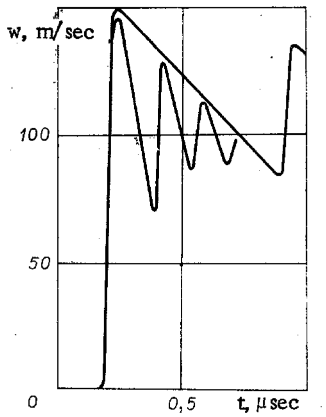

Fig. 5

calculation, of the process in the region of confluence of the trajectories of the change of state along the characteristics. As was to be expected, the variation in the amplitude of the tension wave as it is propagated from the free surface into the interior of the plate or specimen is asymptotic in nature. The failure process can lead to a change not only in the value but also in the sign of the inclination of the trajectories of the change of state along the characteristics C_+ and C_-. At the point of confluence of the trajectories of the change of state along the characteristics C_+, the calculation indicates that they are tangent to the trajectories of the change of state along C_- and along k = const, and the inclination of the tangent is close to the ventrical $\frac{\partial u}{\partial t} = \frac{du}{dt}\Big|_{C_+} = \frac{du}{dt}\Big|_{C_-} \approx 0$. In accordance with (4), (6), (7), it follows from this that at the point of confluence

$$\frac{\partial v}{\partial t} = 0, \quad \frac{\partial p}{\partial t} = \rho_0^2 a^2 \frac{\partial v_{cr}}{\partial t}. \tag{14}$$

Qualitatively the wave propagating in the positive direction from the failure zone can be represented as the superposition of two waves – the unloading part of the incident pulse and the compression wave appearing as a result of the failure. The minimum on the profile of w(t) in this case can be treated as equality in absolute magnitude of the values of $\dot{p}_0$ in the incident rarefaction wave and in the compression wave resulting from the failure. Taking account of (14), we can get from this some idea of the primary estimate for the failure kinetics (1) on the basis of a series of profiles of w(t) obtained in experiments with different rates of pressure drop in the incident pulse.

The model calculations show that the kinetics of ductile growth of cracks (11), (12) can ensure qualitative similarity between the calculated and experimental profiles of w(t) and functional dependence on the characteristic length of the incident pulse of the value of the drop in w from the maximum to the first minimum. However, in the calculation using these kinetics, we cannot reproduce multiple spalling [1], the failure zone is found to be more blurred than the zone observed experimentally [6], and the amplitude of the spall pulse is reduced as the steepness of the incident pulse increases. Using the function $v_{cr0}(p)$ with saturation in the form (13) aggravates these deficiencies. A comparison with the experimental data leads to the assumption that in reality the failure process takes place more rapidly and sharply and that $\dot{v}_{cr}$ must depend more strongly on v_{cr}. At the same time it would be desirable to find a single failure kinetics that would reconcile the results of the investigations of spall failure and tests on the durability of specimens under load [11, 13]. To satisfy these requirements, we can use a failure kinetics in the form

$$\dot{v}_{cr} = -sp \exp\ (p/p_k + v_{cr}/v_k), \tag{15}$$

where s, p_k, v_k are constants of the material. For constant loading the failure time up to a given value of v_{cr} and the maximum failure time are obtained by integrating (15):

$$\tau = -v_k[1 - \exp\ (-v_{cr}/v_k)]\ \exp\ (-p/p_k)/sp,$$
$$\tau_{max} = -v_k \exp\ (-p/p_k)/sp,$$

where $p < 0$. Thus, (15) yields a nearly exponential relation for the durability of the specimen under load as a function of the applied tensile stress, as is observed experimentally. Examples of profiles calculated in the hydrodynamic approximation, using (15), where $s = 10^{-9}$ $cm^4 \cdot g^{-2} \cdot sec$, $p_k = -3 \cdot 10^9$ $g \cdot cm^{-1} \cdot sec^{-2}$, $v_k = 6 \cdot 10^{-5}$ $cm^3 \cdot g^{-1}$, are shown in Fig. 5. As was to be expected, the use of the kinetic relation (15) led to an increase in the amplitude of the spall pulse and a narrowing of the failure zone. We also observe multiple spalling.

Thus, the idea of a continuous process of failure, using the specific volume of the cracks as the measure of failure, can ensure a description of the results of investigations of spall phenomena by comparing the data of experiments and model calculations. Analysis shows the presence of a number of gasdynamics properties of the process of spall failure, which can be used to obtain additional experimental information concerning failure kinetics. Thus, it seems entirely possible to use the influence of failure on the law of variation of the amplitude of the tension wave in order to estimate the initial rate of the process. To clarify the specific fundamental form of the failure kinetics (1) of different materials and determine the quantitative characteristics of the kinetics, further experimental and theoretical investigations must be carried out.

LITERATURE CITED

1. B. R. Breed, C. L. Mader, and D. Venabie, "Technique for the determination of dynamic-tensile-strength characteristics," J. Appl. Phys., 38, No. 8 (1967).
2. F. R. Tuler and B. M. Butcher, "A criterion for the time dependence of dynamic fracture," Int. J. Fract. Mech., 4, No. 2 (1968).
3. L. J. Cohen and H. M. Berkowitz, "Time-dependent fracture criteria for 6061-T6 aluminum under stress-wave loading in uniaxial strain," Int. J. Fract. Mech., 7, No. 2 (1971).
4. A. G. Ivanov, "Spalling in the quasiacoustic approximation," Fiz. Goreniya Vzryva, 11, No. 3 (1975).
5. B. A. Tarasov, "Quantitative description of spall damage," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1973).
6. T. W. Barbee, L. Seeman, Jr.,and R. Crewdson, "Dynamic fracture criteria for ductile and brittle metals," J. Mater., 7, No. 3 (1972).
7. L. Davison and A. L. Stevens, "Continuum measures of spall damage," J. Appl. Phys., 43, No. 3 (1972).
8. S. A. Novikov, I. I. Divnov, and A. G. Ivanov, "Investigation of the failure of steel, aluminum, and copper under explosive loading," Fiz. Met. Metalloved., 21, No. 4 (1966).
9. S. Cochran and D. Banner, "Spall studies in uranium," J. Appl. Phys., 48, No. 7 (1977).
10. A. M. Molodets, "Measurements of spall strength of three steels," in: Detonation. Critical Phenomena. Physicochemical Transformations in Shock Waves [in Russian], Chernogolovka (1978).
11. N. A. Zlatin, G. S. Pugachev, S. M. Mochalov, and A. M. Bragov, "Time-dependent laws governing the process of failure of metals under intensive loads," Fiz. Tverd. Tela, 17, No. 9 (1975).
12. A. A. Samarskii and Yu. P. Popov, Difference Schemes for Gasdynamics [in Russian], Nauka, Moscow (1975).
13. V. R. Regel', A. I. Slutsker, and É. I. Tomashevskii, "Kinetic nature of the strength of solids," Usp. Fiz. Nauk, 106, No. 2 (1972).